Indiana Academic Standards

Mathematics: Grades 3-5

I. Introduction

The college and career ready Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The definitions that guided this work were created by the Indiana Education Roundtable, Department of Education, Center for Education & Career innovation, Commission for Higher Education and the Department of Workforce Development. The definition for college and career ready by this group and used throughout this process is as follows: "College-and – career ready means an individual has the knowledge, skills and abilities to succeed in post-secondary education and economically-viable career opportunities." Additionally Public Law 31-2014 [SEA 91] defines college and career readiness educational standards as "the standards that a high school graduate must meet to obtain the requisite knowledge and skill to transition without remediation to post-secondary education or training, and ultimately into a sustainable career."

Standards Process

The Indiana Academic Standards were created through a collaborative process with input from teams of K-12 educators and parents representing school corporations located throughout the state of Indiana; professors of higher education, representing a wide range of Indiana's public and private colleges and universities; and representatives from Indiana businesses and industries. The purpose of the standards process was to design college and career ready standards that would ensure students who complete high school in Indiana are ready for college and careers.

History

Public Law 286 was passed by the Indiana General Assembly in 2013, which created Indiana Code 20-19-2-14.5. The law requires the Indiana State Board of Education to perform a comprehensive review of Indiana's current standards (which were the 2010 Common Core State Standards[1]) and to adopt college and career ready educational standards no later than July 1, 2014.

In the fall of 2013, the Indiana Department of Education established Technical Teams, which were comprised of K-12 educators in English/Language Arts and Mathematics. The Technical Teams were responsible for reviewing the existing Indiana Academic Standards (Common Core State Standards) and providing suggestions for edits and word changes to improve the clarity and progression of the standards. The Department also created Advisory Teams, which were made up of educators from k-12, parents, community members, and higher education institutions across Indiana. The Advisory Teams were responsible for reviewing the work of the Technical Teams and providing additional input.

Evaluation Process

In January of 2014, the Indiana Department of Education, in collaboration with the Indiana State Board of Education, established Evaluation Teams. The Evaluation Teams were responsible for additional layers beyond the work of the Technical and Advisory Teams. The Evaluation Teams were tasked with conducting a comprehensive analysis of several sets of standards, with the goal of identifying the standards that most clearly aligned with the content and skills that Hoosier students would need to know and be able to do in order to be college and career ready.

Membership for the Evaluation Teams was gleaned from individuals who had previously participated on either a Technical Team or an Advisory Team. The Evaluation Team members were selected for their subject matter expertise (in English/Language Arts or Mathematics) and their classroom teaching experience.

The Evaluation Teams were made up of K-12 educators who represented a wide variety of Indiana school corporations with over 445 years of combined classroom teaching experience, and higher education subject matter experts in English/Language Arts and Mathematics, representing Indiana's public and private institutions of higher education.

The Evaluation Teams met for the first time in February of 2014. The English/Language Arts evaluation teams were given the E/LA Common Core State Standards, as well as Indiana's 2006 E/LA Academic Standards and the standards created by the National Council of Teachers of English. The Mathematics evaluation teams were given the Mathematics Common Core State Standards, as well as Indiana's 2000 Math Academic Standards, Indiana's 2009 Math Academic Standards, and the standards created by the National Council of Teachers of Mathematics.

The panel was instructed to independently evaluate each set of standards, identifying whether the standard was wholly aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready; partially aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready; or not aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready. The results of the evaluation were processed according to a forced consensus requirement—a majority requirement was calculated for each group of standards that was reviewed. Any standard that received a fully aligned rating by the majority of reviewers was marked as fully aligned; any standard that received a not aligned rating by the majority of reviewers was marked as not aligned; and any standard that received a partially aligned rating by the majority, or did not have a majority result, was marked as partially aligned.

Once the evaluations were complete, the results were compiled, and the Evaluation Teams were brought together to conduct a consensus process. The consensus process was blind (meaning that the Evaluation Team members did not know the origin of the standards that they were discussing). Through the consensus process, the Evaluation Teams were asked to select the standards that best and most thoroughly represented what students should know and be able to do in various areas of English/Language Arts and Mathematics in order to be college and career ready. The Evaluation Teams selected the standards that they found to be most appropriate; combined standards to create a more appropriate, rigorous, or clear standard; or, if they determined that gaps existed, wrote standards, or reviewed standards from other states (for example, the English/Language Arts Evaluation Teams reviewed the 2010 draft standards from Massachusetts).

Once the Evaluation Teams had selected the standards (from Common Core State Standards, Indiana Academic, or other states) or had written standards where they found gaps, the list of knowledge and skills identified as necessary for students to be college and career ready was posted for public comment.

<u>Public Comment, Public Hearings, and National Expert Review</u>

The draft college and career ready Indiana Academic Standards were posted for the public to review on February 19, 2014. The public was invited to provide comment through March 12. Over 2000 public comments were received. There were also three public hearings, which were held in southern, central, and northern Indiana, to receive public comment on the draft standards.

The comments from both the online public comment and the public hearings were compiled, reviewed and used to contribute to further iterations of the standards.

In addition, a variety of national experts were contacted to review the draft standards posted on February 19. The results of the reviews were discussed, and portions of the reviews were incorporated into further iterations of the standards.

Reconvening of Evaluation Teams

The Evaluation Teams were reconvened in March of 2014. The teams were tasked with incorporating public comment, and l national expert review to ensure that the draft standards were aligned across grade levels and showed appropriate progression from grade to grade. The Evaluation Teams were also tasked with editing and revising standards for clarity, and addressing any other public comments and national expert review around grade appropriateness, bias, embedded pedagogy, or other factors.

Once the Evaluation Teams completed their reviews, the results were sent to the College and Career Ready (CCR) Panels for final review and approval. The results were also shared with additional national experts, who provided reviews. The results of those reviews were analyzed and synthesized and shared with the CCR Panels.

College and Career Ready (CCR) Panels

The College and Career Ready Panels were created in order to ensure that the standards that Indiana developed were aligned with what colleges, universities, industries, and businesses deem necessary for students to be college and career ready. The CCR Panels were made up of subject matter experts from a variety of Indiana public and private colleges and universities, as well as individuals representing Indiana's businesses and industries.

The CCR Panels were brought together in late March of 2014 to review the draft Indiana Academic Standards that had been reviewed and vetted by the Evaluation Teams in mid-March of 2014. The CCR Panels were tasked with reviewing the standards from 12th grade through kindergarten to ensure that the standards were clear and understandable; aligned across grade levels, showing appropriate progression from grade to grade; and designed to prepare students for college and career readiness. The CCR panels met several times throughout the end of March 2014 and early April 2014 to accomplish this task. At their last meeting, the CCR panel members were asked to sign-off on the draft standards, indicating whether, in their professional opinion, the standards were poised to prepare Hoosier students to be college and career ready.

Indiana Academic Standards

The culmination of the efforts of the Technical Teams, Advisory Teams, Evaluation Teams, and CCR Panels is the college and career ready Indiana Academic Standards that are college and career ready. While many of the standards originated from various sources, including the Common Core State Standards; 2000, 2006, and 2009 Indiana Academic Standards; Massachusetts 2010 Draft English/Language Arts Standards; Virginia Standards of Learning; Nebraska English/Language Arts Standards; the National Council of Teachers of Mathematics; and the National Council of Teachers of English, a number of original standards were also written by members of the Evaluation Teams or CCR Panels.

The process was designed to identify the clearest, most rigorous, and best aligned standards in Mathematics and English/Language Arts to ensure that Hoosier students will graduate meeting the definitions for college and career as defined in Indiana's processes.

<u>What are college and career ready Indiana Academic Standards?</u>

The college and career ready Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The Indiana Academic Standards for English/Language Arts demonstrate what students should know and be able to do in the areas of Reading, Writing, Speaking and Listening, and Media Literacy. The Indiana Academic Standards for Mathematics demonstrate what students should know and be able to do in the areas of K-8 Mathematics; Algebra I, II, and Geometry; and higher-level high school Mathematics courses. The Indiana Academic Standards for Content Area Literacy (History/Social Studies and Science/Technical Subjects) indicate ways in which students should be able to incorporate literacy skills into various content areas at the 6-12 grade levels.

What are the college and career ready Indiana Academic Standards NOT?

1). *The standards are not curriculum.*

While the standards may be used as the basis for curriculum, **the college and career ready *Indiana Academic Standards are not a curriculum*.** Therefore, identifying the sequence of instruction at each grade—what will be taught and for how long—requires concerted effort and attention at the corporation and school levels. While the standards may have examples embedded, and resource materials may include guidelines and suggestions, the standards do not prescribe any particular curriculum. Curriculum is determined locally by a corporation or school and is a prescribed learning plan toward educational goals that includes curricular tools and instructional materials, including textbooks, that are selected by the corporation/school and adopted through the local school board.

2). *The standards are not instructional practices.*

While the standards demonstrate what Hoosier students should know and be able to do in order to be prepared for college and careers, the standards are not instructional practices. The educators and subject matter experts that worked on the standards have taken care to ensure that the standards are free from embedded pedagogy and instructional practices. ***The standards do not define <u>how</u> teachers should teach.*** The standards must be complemented by well-developed, aligned, and appropriate curricular materials, as well as robust and effective instructional best practices.

3). *The standards do not necessarily address students who are far below or far above grade-level.*

The standards are designed to show what the average Hoosier student should know and be able to do in order to be prepared for college and career. However, some students may be far below grade level or in need of special education, and other students may be far above grade level. The standards do not provide differentiation or intervention methods necessary to support and meet the needs of these students. It is up to the district, school, and educators to determine the best and most effective mechanisms of standards delivery for these students.

4). *The standards do not cover all aspects of what is necessary for college and career readiness*

While the standards cover what have been identified as essential skills for Hoosier students to be ready for college and careers, the standards are not—and cannot be—an exhaustive list of what students need in order to be ready for life after high school. Students, especially younger students, require a wide range of physical, social, and emotional supports in order to be prepared for the rigors of each educational progression (elementary grades to middle grades; middle grades to high school; and high school to college or career).

II. Acknowledgements

The college and career ready Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, parents higher education professors, and representatives of Indiana business and industry. Additionally, the members of the public, including parents, community members, policymakers, and educators who took time to provide public comments, whether through the online comment tool or in person at the various public hearings, have played a key role in contributing to the Indiana Academic Standards.

The Indiana Department of Education and Indiana State Board of Education would like to thank Ms. Sujie Shin of the Center on Standards and Assessment Implementation for providing expert facilitation throughout the process and acting in an advisory capacity. The Department and Board would also like to thank the individuals and organizations who provided national expert reviews of the draft standards.

We wish to specially acknowledge the members of the Technical Teams, Advisory Teams, Evaluation Teams, and College and Career Ready Panels who dedicated hundreds of hours to the review, evaluation, synthesis, rewriting, and creation of standards designed to be of the highest quality so that our Hoosier students who are ready for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: GRADES 3-5

The Mathematics standards for grades 3-5 are supplemented by the Process Standards for Mathematics.

The Mathematics standards for grades 3-5 are made up of 5 strands: Number Sense; Computation; Algebraic Thinking; Geometry; Measurement; and Data Analysis (note that Data Analysis becomes Data Analysis and Statistics in Grade 5). The skills listed in each strand indicate what students in grades 3-5 should know and be able to do in Mathematics.

NUMBER SENSE

GRADE 3	GRADE 4	GRADE 5
3.NS.1: Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.	**4.NS.1:** Read and write whole numbers up to 1,000,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000,000.	**5.NS.1:** Use a number line to compare and order fractions, mixed numbers, and decimals to thousandths. Write the results using >, =, and < symbols.
3.NS.2: Compare two whole numbers up to 10,000 using >, =, and < symbols.	**4.NS.2:** Compare two whole numbers up to 1,000,000 using >, =, and < symbols.	**5.NS.2:** Explain different interpretations of fractions, including: as parts of a whole, parts of a set, and division of whole numbers by whole numbers.
3.NS.3: Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by *a* parts of size 1/b. [*In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8*.]	**4.NS.3:** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Name and write mixed numbers using objects or pictures. Name and write mixed numbers as improper fractions using objects or pictures.	**5.NS.3:** Recognize the relationship that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and inversely, a digit in one place represents 1/10 of what it represents in the place to its left.
3.NS.4: Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	**4.NS.4:** Explain why a fraction, a/b, is equivalent to a fraction, $(n \times a)/(n \times b)$, by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [*In grade 4, limit denominators of fractions to 2, 3, 4, 5, 6, 8, 10, 25, 100.*]	**5.NS.4:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

	3.NS.5: Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.	**4.NS.5:** Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark, such as 0, 1/2, and 1). Recognize comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	**5.NS.5:** Use place value understanding to round decimal numbers up to thousandths to any given place value.
	3.NS.6: Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.	**4.NS.6:** Write tenths and hundredths in decimal and fraction notations. Use words, models, standard form and expanded form to represent decimal numbers to hundredths. Know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 = 0.50, 7/4 = 1 3/4 = 1.75).	**5.NS.6:** Understand, interpret, and model percents as part of a hundred (e.g. by using pictures, diagrams, and other visual models).
	3.NS.7: Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).	**4.NS.7:** Compare two decimals to hundredths by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	
	3.NS.8: Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	**4.NS.8:** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.	
	3.NS.9: Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.	**4.NS.9:** Use place value understanding to round multi-digit whole numbers to any given place value.	

COMPUTATION

	GRADE 3	GRADE 4	GRADE 5
	3.C.1: Add and subtract whole numbers fluently within 1000.	**4.C.1:** Add and subtract multi-digit whole numbers fluently using a standard algorithmic approach.	**5.C.1:** Multiply multi-digit whole numbers fluently using a standard algorithmic approach.

	3.C.2: Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.	**4.C.2:** Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Describe the strategy and explain the reasoning.	**5.C.2:** Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning used.
	3.C.3: Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.	**4.C.3:** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning.	**5.C.3:** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
	3.C.4: Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).	**4.C.4:** Multiply fluently within 100.	**5.C.4:** Add and subtract fractions with unlike denominators, including mixed numbers.
	3.C.5: Multiply and divide within 100 using strategies, such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.	**4.C.5:** Add and subtract fractions with common denominators. Decompose a fraction into a sum of fractions with common denominators. Understand addition and subtraction of fractions as combining and separating parts referring to the same whole.	**5.C.5:** Use visual fraction models and numbers to multiply a fraction by a fraction or a whole number.
	3.C.6: Demonstrate fluency with multiplication facts and corresponding division facts of 0 to 10.	**4.C.6:** Add and subtract mixed numbers with common denominators (e.g. by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction).	**5.C.6:** Explain why multiplying a number by a fraction greater than 1 results in a product greater than the given number. Explain why multiplying a number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence, a/b = (n × a)/(n × b), to the effect of multiplying a/b by 1.
		4.C.7: Show how the order in which two numbers are multiplied (commutative property) and how numbers are grouped in multiplication (associative property) will not change the product. Use these properties to show that numbers can by multiplied in any order. Understand and use the distributive property.	**5.C.7:** Use visual fraction models and numbers to divide a unit fraction by a non-zero whole number and to divide a whole number by a unit fraction.

			5.C.8: Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning.
			5.C.9: Evaluate expressions with parentheses or brackets involving whole numbers using the commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property.

ALGEBRAIC THINKING

	GRADE 3	GRADE 4	GRADE 5
	3.AT.1: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	**4.AT.1:** Solve real-world problems involving addition and subtraction of multi-digit whole numbers (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	**5.AT.1:** Solve real-world problems involving multiplication and division of whole numbers (e.g. by using equations to represent the problem). In division problems that involve a remainder, explain how the remainder affects the solution to the problem.
	3.AT.2: Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	**4.AT.2:** Recognize and apply the relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve real-world and other mathematical problems.	**5.AT.2:** Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models and equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess whether the answer is reasonable.
	3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	**4.AT.3:** Interpret a multiplication equation as a comparison (e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7, and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.	**5.AT.3:** Solve real-world problems involving multiplication of fractions, including mixed numbers (e.g., by using visual fraction models and equations to represent the problem).

	3.AT.4: Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.	**4.AT.4:** Solve real-world problems with whole numbers involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison. [In grade 4, division problems should not include a remainder.]	**5.AT.4:** Solve real-world problems involving division of unit fractions by non-zero whole numbers, and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem).
	3.AT.5: Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	**4.AT.5:** Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having common denominators (e.g., by using visual fraction models and equations to represent the problem).	**5.AT.5:** Solve real-world problems involving addition, subtraction, mutliplication, and division with decimals to hundredths, including problems that involve money in decimal notation (e.g. by using equations to represent the problem).
	3.AT.6: Create, extend, and give an appropriate rule for number patterns using multiplication within 1000.	**4.AT.6:** Understand that an equation, such as y = 3x + 5, is a rule to describe a relationship between two variables and can be used to find a second number when a first number is given. Generate a number pattern that follows a given rule.	**5.AT.6:** Graph points with whole number coordinates on a coordinate plane. Explain how the coordinates relate the point as the distance from the origin on each axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
			5.AT.7: Represent real-world problems and equations by graphing ordered pairs in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
			5.AT.8: Define and use up to two variables to write linear expressions that arise from real-world problems, and evaluate them for given values.

GEOMETRY

	GRADE 3	GRADE 4	GRADE 5
	3.G.1: Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.	**4.G.1:** Identify, describe, and draw parallelograms, rhombuses, and trapezoids using appropriate tools (e.g., ruler, straightedge and technology).	**5.G.1:** Identify, describe, and draw triangles (right, acute, obtuse) and circles using appropriate tools (e.g., ruler or straightedge, compass and technology). Understand the relationship between radius and diameter.

	3.G.2: Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.	**4.G.2:** Recognize and draw lines of symmetry in two-dimensional figures. Identify figures that have lines of symmetry.	**5.G.2:** Identify and classify polygons including quadrilaterals, pentagons, hexagons, and triangles (equilateral, isosceles, scalene, right, acute and obtuse) based on angle measures and sides. Classify polygons in a hierarchy based on properties.
	3.G.3: Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.	**4.G.3:** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.	
	3.G.4: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).	**4.G.4:** Identify, describe, and draw rays, angles (right, acute, obtuse), and perpendicular and parallel lines using appropriate tools (e.g., ruler, straightedge and technology). Identify these in two-dimensional figures.	
		4.G.5: Classify triangles and quadrilaterals based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles (right, acute, obtuse).	

MEASUREMENT

	GRADE 3	GRADE 4	GRADE 5
	3.M.1: Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).	**4.M.1:** Measure length to the nearest quarter-inch, eighth-inch, and millimeter.	**5.M.1:** Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step real-world problems.
	3.M.2: Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.	**4.M.2:** Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Express measurements in a larger unit in terms of a smaller unit within a single system of measurement. Record measurement equivalents in a two-column table.	**5.M.2:** Find the area of a rectangle with fractional side lengths by modeling with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

	3.M.3: Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.	**4.M.3:** Use the four operations (addition, subtraction, multiplication and division) to solve real-world problems involving distances, intervals of time, volumes, masses of objects, and money. Include addition and subtraction problems involving simple fractions and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	**5.M.3:** Develop and use formulas for the area of triangles, parallelograms and trapezoids. Solve real-world and other mathematical problems that involve perimeter and area of triangles, parallelograms and trapezoids, using appropriate units for measures.
	3.M.4: Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase.	**4.M.4:** Apply the area and perimeter formulas for rectangles to solve real-world problems and other mathematical problems involving shapes. Recognize area as additive and find the area of complex shapes composed of rectangles by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts; apply this technique to solve real-world problems and other mathematical problems involving shapes.	**5.M.4:** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.
	3.M.5: Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.	**4.M.5:** Understand that an angle is measured with reference to a circle, with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure other angles. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.	**5.M.5:** Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for right rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve real-world problems and other mathematical problems involving shapes.
	3.M.6: Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	**4.M.6:** Measure angles in whole-number degrees using appropriate tools. Sketch angles of specified measure.	**5.M.6:** Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems and other mathematical problems.
	3.M.7: Find perimeters of polygons given the side lengths or by finding an unknown side length.		

DATA ANALYSIS (DATA ANALYSIS AND STATISTICS FOR GRADE 5)

	GRADE 3	GRADE 4	GRADE 5
	3.DA.1: Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step "how many more" and "how many less" problems regarding the data and make predictions based on the data.	**4.DA.1:** Formulate questions that can be addressed with data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, and bar graphs.	**5.DS.1:** Formulate questions that can be addressed with data and make predictions about the data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, bar graphs, and line graphs. Recognize the differences in representing categorical and numerical data.
	3.DA.2: Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.	**4.DA.2:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using data displayed in line plots.	**5.DS.2:** Understand and use measures of center (mean and median) and frequency (mode) to describe a data set.
		4.DA.3: Interpret data displayed in a circle graph.	