Indiana Department of Education

Indiana Academic Standards
Mathematics: Calculus

Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards have been developed through the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their

	limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Calculus

Limits and Continuity	
C.LC.1	Understand the concept of limit and estimate limits from graphs and tables of values.
C.LC.2	Find limits by substitution.
C.LC.3	Find limits of sums, differences, products, and quotients.
C.LC.4	Find limits of rational functions that are undefined at a point.
C.LC.5	Find limits at infinity.
C.LC.6	Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. Find special limits.
C.LC.7	Find one-sided limits.

C.LC.8	Understand continuity in terms of limits.
C.LC.9	Decide if a function is continuous at a point.
C.LC.10	Find the types of discontinuities of a function.
C.LC.11	Understand and use the Intermediate Value Theorem on a function over a closed interval.
C.LC.12	Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval.

Differentiation	
C.D.1	Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change.
C.D.2	State, understand, and apply the definition of derivative.
C.D.3	Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.
C.D.4	Find the derivatives of sums, products, and quotients.
C.D.5	Find the derivatives of composite functions, using the chain rule.
C.D.6	Find the derivatives of implicitly-defined functions.
C.D.7	Find the derivatives of inverse functions.
C.D.8	Find second derivatives and derivatives of higher order.

C.D.9	Find derivatives using logarithmic differentiation.
C.D.10	Understand and apply the relationship between differentiability and continuity.
C.D.11	Understand and apply the Mean Value Theorem.

Applications of Derivatives	
C.AD.1	Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents.
C.AD.2	Find a tangent line to a curve at a point and a local linear approximation.
C.AD.3	Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'.
C.AD.4	Solve real-world and other mathematical problems finding local and absolute maximum and minimum points with and without technology.
C.AD.5	Analyze real-world problems modeled by curves, including the notions of monotonicity and concavity with and without technology.
C.AD.6	Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f''. Understand points of inflection as places where concavity changes.
C.AD.7	Use first and second derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology. Compare the corresponding characteristics of the graphs of f, f', and f''.
C.AD.8	Use implicit differentiation to find the derivative of an inverse function.

C.AD.9	Solve optimization real-world problems with and without technology.
C.AD.10	Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including distance, velocity, and acceleration.
C.AD.11	Find the velocity and acceleration of a particle moving in a straight line.
C.AD.12	Model rates of change, including related rates problems.

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Integrals	
C.I.1	Use rectangle approximations to find approximate values of integrals.
C.I.2	Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.
C.I.3	Interpret a definite integral as a limit of Riemann Sums.
C.I.4	Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval.
C.I.5	Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.
C.I.6	Understand and use these properties of definite integrals. a. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ b. $\int_a^b k \cdot [f(x)]dx = k \cdot \int_a^b f(x)dx$ c. $\int_a^a f(x)dx = 0$ d. $\int_a^b f(x)dx = -\int_b^a f(x)dx$ e. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

	f. $If\ f(x) \leq g(x)\ on\ [a,b], then\ \int_a^b f(x)dx \leq \int_a^b g(x)dx$
C.I.7	Understand and use integration by substitution (or change of variable) to find values of integrals.
C.I.8	Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

Applications of Integrals	
C.AI.1	Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.
C.AI.2	Solve separable differential equations and use them in modeling real-world problems with and without technology.
C.AI.3	Solve differential equations of the form y' = ky as applied to growth and decay problems.
C.AI.4	Use definite integrals to find the area between a curve and the x-axis, or between two curves.
C.AI.5	Use definite integrals to find the average value of a function over a closed interval.
C.AI.6	Use definite integrals to find the volume of a solid with known cross-sectional area.
C.AI.7	Apply integration to model and solve (with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.